Dynamic Symmetry---A Criticism

By Edwin M. Blake

Mr. Jay Hambidge of Boston and New York has for a number of years been engaged in a study of the proportions found, on the one hand, in the artistic designs of the early Egyptians and the Greeks, and on the other, in plant forms and in the human skeleton. The proportions thus obtained, with few exceptions, Mr. Hambidge finds belong to one of three systems according as they depend on the square root of two, of three, or of five. The human skeleton and most Greek vases and other objects of art of the best period are designed according to the root-five system. These three root systems, especially the latter, are conceived to be expressive of life, growth, and vitality and accordingly the term "dynamic symmetry" has been applied to them. During the decadence of Greek art the principles and methods of dynamic symmetry were lost, and hence much to the detriment of art were not used during the Roman period, the Middle Ages, nor since, until the labors of Mr. Hambidge led to their rediscovery. Art since the Greeks, it is maintained, is characterized by having "static symmetry," the dimensions being commensurable one to another, and thereby lacks the vitality and subtlety of form which the earlier art possessed.

Dynamic symmetry has been made known to the public by various lectures before art societies, and at museums and universities both here and abroad, also through *The Diagonal*[1], a monthly magazine of which Mr. Hambidge is editor, and his monograph: *Dynamic Symmetry: The Greek Vase.*[2] In the prosecution of his investigations the author has received encouragement and material assistance from two of our leading universities and from two of our largest art museums. The program for developing the subject looks not alone to the past. If the Greeks could

[1] Published by the Yale University Press, beginning November 1919. References to volume I of this will be abbreviated: "D".
[2] Yale University Press, 1920. Abbreviated for reference: "G. V."

apply dynamic symmetry to design with such signal success, may not we, now that its methods are again available, introduce into art those subtleties of proportion, those vitalizing forms, which, we are told, have so long been absent? To this end lectures have been given to classes of designers in New York and Boston, and as a consequence one of New York's leading jewelers is advertising silverware made according to the method, and an art school is using it in figure composition. In *The Diagonal* are to be found letters of commendation and appreciative reviews of dynamic symmetry attesting to the enthusiasm which it has aroused in many quarters.

Your reviewer has three times heard Mr. Hambidge lecture, has conversed with him and some of his co-workers, has read the magazine and the monograph with considerable care, and in addition has made independent studies of systems of rectangles, and of individual rectangles possessing special properties—all of which has led him to the conclusion that in spite of the enthusiasm with which dynamic symmetry has been received, there is little ground to support the claims made for it. It is very doubtful whether the Greeks ever used dynamic symmetry, and whether in the absence of documentary evidence, it is now possible by measurement to prove either that they did or did not, or to differentiate static from dynamic. There is more than a doubt that dynamic symmetry, in its applications to design, introduces anything of aesthetic value; nor is it possible, we believe, to substantiate a distinct difference of artistic quality and superiority on the basis of the systems of rectangles involved. The author's attempt to support his conclusions by measurement of a human skeleton virtually assumes what it is proposed to prove.

The Hambidge writings are more difficult reading than their subject matter warrants. There is an almost entire absence of clearly stated definitions of concepts and terms. From the description of a figure it is often difficult to determine the order of constructing the several lines and to separate what is assumed from what is to be shown; conclusions are reached for which there is no justification in the argument and out of several solutions,

one alone may be selected.[1] With one or two exceptions in *The Diagonal*, the dimensions of the vases and other objects analyzed are never given, and hence it is impossible to ascertain how closely the diagrams given fit the originals, nor is there any discussion of the probable discrepancies between the ideal design as it was in the mind of its Greek creator, and the finished vase as it now stands. Sweeping claims are made for the artistic excellence which has and can be achieved by the use of dynamic symmetry but not a word from modern psychology nor an experiment in support of them.

Analysis by Systems of Rectangles.

We will now briefly go over the Hambidge procedure in analyzing designs and state the claims made for it. The shape of any rectangle may be expressed by stating the quotient obtained by dividing one of its longer sides by a shorter, the resulting ratio being greater than one; or by dividing a shorter by a longer giving a ratio less than one. The product of the two ratios is unity. Both ratios of the square have the value one. Rectangles differing in size but having equal ratios are similar, and a well known construction enables one to draw a rectangle, similar to a given one, whose longer or shorter sides shall have a prescribed length. The lengths of the two diagonals of any rectangle are equal, each being the square root of the sum of the squares of two adjacent sides. The diagonal of a square whose side is unity has length $\sqrt{2}$, and if a rectangle be constructed by taking for adjacent sides the side of a square and its diagonal, its ratio will be $\sqrt{2}$. This is called a "root-two rectangle." The shorter side of this rectangle and its diagonal, if used to construct a second rectangle, give the "root-three rectangle." The diagonal of the rectangle formed by placing two squares side by side has length $\sqrt{5}$, and the rectangle whose sides are unity and $\sqrt{5}$ is the "root-five rectangle." The "root-five system of rectangles," the principal one used in dynamic symmetry, consists, as built up by our author, of the square, the "rectangle of the whirling squares" (G. V. pp. 17, 18), certain rectangles defined by special constructions (G. V. pp. 20-22, 30-39), and combinations of these. By these

[1] See review by a member of the Department of Mathematics of Columbia University, *The Nation*, September 18, 1920, Vol. CXI, pp. 326-327.

means a variety of related rectangles is obtained and the number may be added to, as required, by building up others from those already on hand. The root-two and root-three systems are similarly obtained though these have received less attention.

The Hambidge procedure for the analysis of a Greek vase design, after a careful drawing of a side view (as Fig 1. See D. pp. 116-117) has been made, consists of two steps.

I.—The determination of the enclosing rectangle. The width of the kylix including handles divided by the height gives the ratio 3.115. A rectangle of the root-five system, already on hand, has the ratio 3.118. This is assumed to be the rectangle used by the designer, and is taken for the enclosing rectangle RW, Fig 1. Were no rectangle on hand having a ratio near that of the vase, an attempt would be made to build it up in one of the three systems.

II.—Analysis within the enclosing rectangle to locate details of the design. The processes involved are of two types.

A.—The subdivision of the enclosing rectangle into lesser rectangles which exactly fill it. Thus for the kylix, Fig. 1, the width of the bowl divided by the total height gives the ratio 2.4754, and it is assumed the rectangle whose ratio is 2.472 was used by the designer. This is the rectangle AB. In like manner AC consists of four squares, and the vertical sides of these are prolonged downward to the base of the figure.

B.—The location of further details by the drawing of diagonals. Thus in Fig. 1 the diagonal AP intersects VC at D and the perpendicular through this determines the width of the foot. In other cases the detail may be located by drawing a horizontal or vertical line through the point of intersection of two diagonals (G. V. Fig. 13, p. 52).

The various statements with reference to what analyses of this kind show may conveniently be grouped under four claims.

First Claim. The designers of these objects understood the systems of root rectangles and used them in laying out their designs—the diagrams shown are the actual procedure followed by the designer.

"This symmetry is identical with that used by Greek masters in almost all the art produced during the great

Fig. 1

See D. pp. 116-117

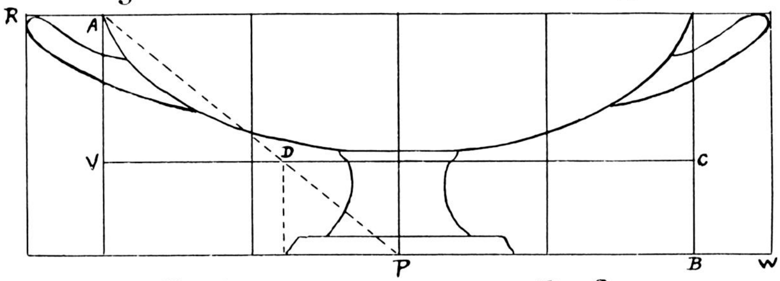


Fig. 2

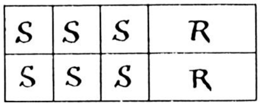


Fig. 3

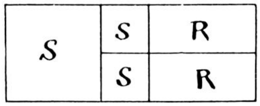


Fig. 4

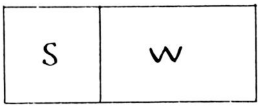


Fig. 5

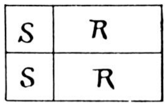


Fig. 6

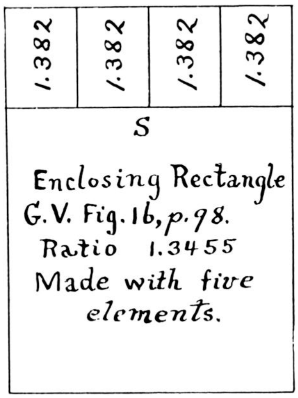


Fig. 7

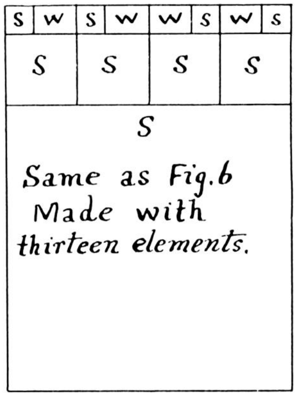


Fig. 8 ($\sqrt{5}$ Sys.)

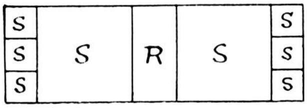
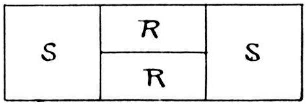


Fig. 9 ($\sqrt{5}$ Sys.)



classical period" (D. p. 1). "At some time during the sixth or seventh century B. C. the Greeks obtained from Egypt knowledge of this manner of correlating elements of design. In their hands it was highly perfected as a practical geometry, and for about three hundred years it provided the basic principle of design for what the writer considers the finest art of the Classic period. . . . Its recovery has given us dynamic symmetry. . . ." (G. V. p. 8) (Compare G. V. pp. 88, 95).

Second Claim. Not all designs possess dynamic symmetry, that is, are capable of being analyzed by the systems of root rectangles.

"Saracenic, Mahomedan, Chinese, Japanese, Persian, Hindu, Assyrian, Coptic, Byzantine, or Gothic art analysis show unmistakably the conscious use of plan schemes and all belong to the same type. Greek and Egyptian art analyses show an unmistakable use of plan schemes of another type. There is no question as to the relative merit of the two types. The latter is immeasurably superior to the former" (D. p. 1) (Also D. p. 10, G. V. p. 7). "The symmetry of the human figure in art since the first century B. C. is undoubtedly static" (G. V. Note VI, p. 157). "The symmetry of all art since Greek classic times is static" (D. p. 27) (See also a comparison of two vases D. p. 53, and a designer exhibiting the transition from static to dynamic, D. p. 104).

Third Claim. The superior value of dynamic symmetry as a method of artistic design resides in the fact that it is nature's principal design scheme as well, as exemplified by plant forms and by the human skeleton. The geometrical property to which this is especially ascribed is that, whereas the lines of the diagram are incommensurable one to another, they are "commensurable in square"—the areas are commensurable.

"The square and the diagonal to its half furnish the series of remarkable shapes which constitute the architectural plan of the plant and the human figure. . . . The Greeks, however, said that such lines [one and the square root of five] were not irrational, because they were commensurable or measurable in square. This is really the great secret of Greek design. In understanding this measurableness of area instead of line the Greek artists

had command of an infinity of beautiful shapes which modern artists are unable to use" (D. p. 14). "Both nature and Greek art show that the measurableness of symmetry is that of *area* and not *line*. . . . This is the secret. Dynamic Symmetry deals with commensurable areas" (D. p. 48) (See also G. V. p. 30; Note III p. 145; Note VI, p. 157, D. p. 45).

Fourth Claim. Since the art of the Greeks is the highest type of art yet created, and since it owes its superiority to the employment of dynamic symmetry in its design, that method—now that it is again available after having been lost for over two thousand years—should be employed by designers if modern design is to attain a high degree of artistic excellence.

"The high standard of perfection in Greek art has always had a depressing effect upon artists who have studied it. . . . It has come to be accepted as beyond human power. . . ." (D. p. 49). "Greek pottery is one of the greatest design fabrics ever created. It is an artistic miracle" (G. V. Note I, p. 143). "If this knowledge [of dynamic symmetry] had not become lost artists today would, undoubtedly, have been creating masterpieces of statuary, painting and architecture equalling or surpassing the masterpieces of the Greek classic age" (G. V. Note VI, p. 157). "The discovery of Dynamic Symmetry places the human skeleton in a new position in relation to art. . . . We must now regard this framework of bone as the principal source of the most vital principles of design" (D. p. 34). "The symmetry of the human figure is dynamic, therefore the selecting of a dynamic rectangle for the purpose of correlating the units of a figure composition is most appropriate." (D. p. 136. See also diagram D. p. 121).

The First and Second Claims.

History tells us that certain rules for mensuration and practices in surveying were early discovered and used by the Egyptians, and by them passed on to the Greeks who beginning with Thales of Miletus about 600 B. C. developed during the following three centuries that body of geometric propositions and demonstrations which have come down to us as Euclid's Elements. The necessary

knowledge of geometry was at hand, but aside from this our author has produced no documentary evidence that dynamic symmetry was ever used. The reference to the "Canon of Polykleitos" and that from Vitruvius (D. pp. 5, 27, 48; G. V. p. 9 and Note III, p. 145) can hardly be taken seriously—any conclusions might be drawn from them. For a subject which was as specific in its rules and applications and as æsthetically significant as dynamic symmetry is represented to have been, which must have become known to all Greek architects and designers of any note for about three centuries—to say nothing of Egypt—and which moreover is even credited with being the very source and inspiration of Greek geometry itself (D. pp. 33, 106; G. V. p. 8)—it is indeed remarkable that no account of the discovery has survived. It is recorded in connection with the sister art, music, that experiments were made by Pythagoras to determine the ratios of the lengths of strings giving the octave and fifth (Ball, *A Short Account of the History of Mathematics*, Second Edition, 1893, p. 23; Cantor, *Vorlesungen über die Geschichte der Mathematik*, Vol. I, p. 153.).

Failing then of documentary support, the whole structure of dynamic symmetry, as applied to works of art, has been built up on these two circumstances: first, Greek geometry was at hand; second, numerous figures constructed by our author, composed of related rectangles and their diagonals, when superposed on drawings of Greek designs coincide, in part and more or less accurately, with the overall dimensions of the design and with several other of its features. In order to show how little such coincidences are capable of indicating the way in which a designer may have worked, we will endeavor to show how very great is the number and how varied the proportions of the figures which Greek geometry has to offer, once it is assumed that Greek designers worked with rectangles and had the necessary geometrical intelligence and ingenuity to use them.

It can easily be shown that the ratios of all rectangles of the root-five system that can be constructed by the various methods of the Hambidge analyses, including those resulting from the drawing of horizontal and vertical lines through the intersections of diagonals, are expressed by

the formula $(a+b\sqrt{5})\div c$, the coefficients a, b, c being positive integers or a or b may be zero. Further, no matter what positive integers are selected for a, b, c, the formula always represents the ratio of a rectangle of the root-five system, and all such rectangles can be readily constructed by a uniform method employing equal squares and equal root-five rectangles, the shorter sides of the latter being equal to the sides of the former. Fig. 2 shows the construction for $(3+\sqrt{5})\div 2$. Let us now suppose we have measured a Greek vase and found the enclosing rectangle to have its ratio not less than 2.715 nor greater than 2.718. If we are not restricted as to the number of squares and root rectangles we may use, it is always possible to build up a rectangle whose ratio shall be within the specified limits, but further, two, ten, or a hundred such rectangles, differing slightly one from another, may likewise be constructed. There is no enclosing rectangle, no matter what its ratio, that cannot thus be constructed many times over. Coming now within the enclosing rectangle, if we are again not restricted, any details may be located and any design whatsoever analyzed. This is not all—like conclusions hold for systems of rectangles based on the square roots of two, three, seven, eleven, thirteen, etc., and for the rational system built on the square alone. It would be unfair to our author to assume that he does not intend some restrictions to be observed though the subject is not mentioned. We will endeavor to show, however, that even observing such limitations as the published analyses indicate, there is still considerable probability that any design whatsoever (of a symmetric or generally rectangular character) may be closely approximated by root-five diagrams, by those of any one of the other root systems, and by those of a rational system.

In building up an enclosing rectangle it is not necessary to use squares and root-rectangles of one size only as was done just above, nor even to confine ourselves to the square S and the root-rectangle R, others may be used as "elementary rectangles" or "elements" along with them. Thus, the rectangle Fig. 2 may be constructed as in Fig. 3 with squares of two sizes, or as in Fig. 4 by a square and a "rectangle of the whirling squares" W. The latter is defined in terms of S and R by the construction Fig.

5. Mr. Hambidge usually uses S, R, and W as elements in building up rectangles, though others are not infrequently employed, especially those having the ratios 1.1708 and 1.382. And now how many elementary rectangles may be used in building up enclosing rectangles? Seven elements are employed in G. V. Fig. 16-3, p. 89; Fig. 2, p. 124; ten elements in G. V. Fig. 7, p. 81; eleven in the ground plan of the Erechtheum (D. p. 70); twelve in G. V. Fig. 4, p. 117; five in G. V. Fig. 16, p. 98 if 1.382 is considered to be an elementary rectangle, otherwise thirteen, see Figs. 6, 7. The ground plan of the Parthenon (G. V. p. 96) requires twelve elements when 1.382 is an element, otherwise thirty-two. The front elevation of the second skeleton (D. pp. 119-120) is composed of twelve elementary rectangles if 1.1708 is an element, but when composed of S, R, W, only, twenty-eight are used. The enclosing rectangle of the front elevation of the torso, under like conditions, is built up of one hundred twenty-two or two hundred forty-two. In view of these precedents we think it not unfair to take for elementary rectangles in any system the square, the root-rectangle, and one or two others; and to employ any number of elements not to exceed twelve in constructing enclosing rectangles.

In order to ascertain what can be done with systems of rectangles a study of three was made: the root-five system with S, R, and W as elements, the root-thirteen system, and a rational system. The elements selected for the second were the square S, the root-thirteen rectangle R, the Q-rectangle whose ratio is $(5+\sqrt{13})\div 3=2.8685$, and the T-rectangle whose ratio is $(\sqrt{13}+1)\div 2=2.3028$. In addition to the square S, the elementary rectangles selected for the rational system were the E-rectangle having the ratio three-halves, the F-rectangle ratio, five-thirds, and the G-rectangle, ratio five-halves. In each of these systems groups of two, three, and in some cases four elements were built up, and both ratios of the resulting rectangles calculated. The last mentioned rectangles can again be combined, two, three, or four at a time to produce rectangles involving not more than twelve elementary rectangles. The method was applied to determining rectangles (with component elements symmetrically arranged) having ratios approximating 3.1148, 2.4754, the square

root of two (1.41421), and the square root of three (1.73205), and resulted as in the following table.

Root-five.	Root-thirteen.	Rational.

Approximating 3.1148.

3.1110	3.1133 (Fig. 10)	3.1108
3.1112	3.1134	3.1111
3.1136	3.1140	3.1118
3.1139 (Fig. 8)	3.1142 (Fig. 11)	3.1122
3.1178	3.1150	3.1128
3.1180 (Fig. 9)	3.1153	3.1138
3.1183	3.1160	3.1143
	3.1161	3.1154 (Fig. 12)
	3.1162	3.1167 (Fig. 13)
	3.1173	
	3.1178	

Approximating 2.4754.

2.4716	2.4716	2.4690
2.4721 (Fig. 14)	2.4726	2.4727
2.4746	2.4743	2.4738
2.4760	2.4756	2.4741
2.4763	2.4758	2.4750 (Fig. 16)
2.4798	2.4763 (Fig. 15)	2.4762
	2.4771	2.4777
	2.4773	2.4788
	2.4779	
	2.4785	
	2.4793	
1.4133	1.4143 (Fig. 18)	1.4134
1.4138	1.4159	1.4143 (Fig. 19)
1.4140 (Fig. 17)	1.4160	1.4146
1.4146		1.4147
1.4149		

Approximating 1.41421.

Approximating 1.73205.

1.7298	1.7300	1.7308
1.7318	1.7305	1.7318
1.7323	1.7307	1.7323
1.7333	1.7311	1.7333
	1.7331	1.7336

The method used in obtaining these results is quite laborious and in no case was a complete search made; there are probably some few additional approximations to the first two ratios. The examination was much less complete for the last two and undoubtedly several more could be found. From these examples, which as far as we know are representative, and from experience gained in using the method, we conclude that any arbitrarily selected ratio can be closely approximated in any one of these systems. Were all the rectangles of a system composed of not more than twelve symmetrically arranged elements tabulated, their number would amount to many thousands, and when the ratios did not exceed ten they would probably not differ on the average more than one thousandth of a unit.

It being evident then that the enclosing rectangle of any design may be determined in either one of the systems, we pass to the subdivision of that rectangle into lesser ones (Step II, A above), which we are told should belong to the same system as the enclosing rectangle (D. p. 13). Among the several devices employed for this purpose the following may be mentioned. (a) Elements by which the enclosing rectangle were defined are employed as subdivisions (G. V. Fig. 4, p. 117). Several arrangements of the elements are usually possible. Frequently a number of more or less distinct combinations of elements serve to define the same enclosing rectangle. (b) The enclosing rectangle may be subdivided into four, nine, or sixteen rectangles similar to itself, each of which may then be subdivided (if desired) into the elements defining the enclosing rectangle. The third of the diagrams given in G. V. Fig. 10, p. 84, consisting of sixteen squares and sixteen root-five rectangles, illustrates this. (c) A vertical line L may be drawn across an enclosing rectangle so that the portion lying to the left of it may be an elementary

rectangle or a simple combination of rectangles. For example, if the enclosing rectangle is of the root-five system and one unit in height, and the line L cuts off a column of one to three elements, there will be a choice of thirty-seven positions for L distant 0.149 to 1.118 from the left end of the enclosing rectangle. The same may be done on the right side, at the top, or bottom. Rectangles thus cut off not infrequently overlap, see G. V. Fig. 4, p. 66. Other rectangles thus created may be resolved into their elements (G. V. Figs. 10, 11, p. 69). (d) Lesser rectangles obtained by the above methods may again be subdivided, and these sometimes into still smaller (G. V. Fig. 15, p. 73; Fig. 2, p. 76). Fig. 5 shows a way of subdividing the rectangle of the whirling squares into two squares and two root-five rectangles, and a root-five rectangle is equal to a square and two rectangles of the whirling squares. There are many like relations in all of the root systems. (e) Two and even three different methods of subdividing the enclosing rectangle may be superposed to effect the analysis (G. V. Figs. 14, b, c, p. 87; Fig. 16, p. 89; Figs. 5, 7, p. 107). (f) When the handles of a vase extend beyond the bowl, a rectangle enclosing the bowl is frequently treated as the principal rectangle of the design, the analysis being directed to it rather than to the enclosing rectangle. Thus an important option is introduced (G. V. Fig. 18, p. 99; Fig. 19, p. 100). A rectangle passing through the middle portion of the diagram sometimes determines width of lip or foot (G. V. Fig. 16, p. 98. Both lip and foot. Note proximity of bowl to enclosing rectangle).

Any one who will take the trouble to go over these several methods of subdividing an enclosing rectangle, with a view to estimating the number of lines which they permit drawing across it, will, we believe, become convinced that the number is very great and that they form a fine net-work over the whole. It seems hardly possible that any detail would escape had one only time and patience to try all variations—if indeed the task were not practically endless. However, one may always fall back on the drawing of diagonals to complete an analysis (Step II, B above). A diagram will serve to show the potency of this procedure. Fig. 20 was obtained from G. V. Fig. 8, p. 95 by

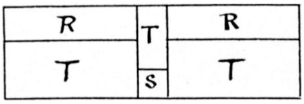


Fig. 10 ($\sqrt{13}$ Sys.)

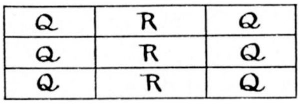


Fig. 11 ($\sqrt{13}$ Sys.)

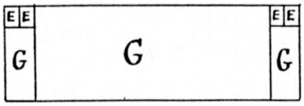


Fig. 12 (Rational Sys.)

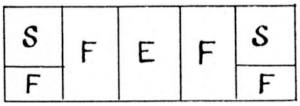


Fig. 13 (Rational Sys.)

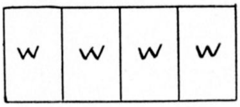


Fig. 14 ($\sqrt{5}$ Sys.)

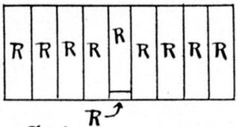


Fig. 15 ($\sqrt{13}$ Sys.)

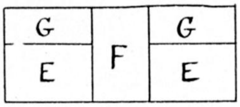


Fig. 16 (Rat. Sys.)

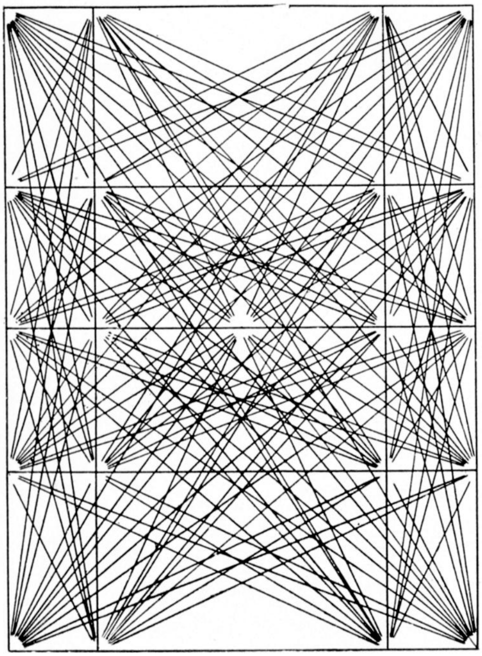


Fig. 20 See G.V. Fig 8, p. 95

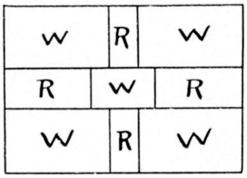


Fig. 17 ($\sqrt{5}$ Sys.)

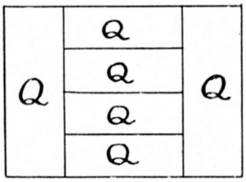


Fig. 18 ($\sqrt{13}$ Sys.)

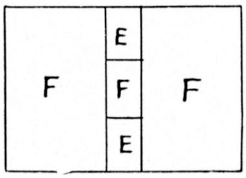


Fig. 19 (Rat. Sys.)

drawing diagonals through six points within, the vertices of the enclosing rectangle, three points on each side, and two at top and bottom. It will be remembered that to locate a detail a perpendicular or horizontal line may be drawn through the intersection of any two diagonals. Another subdivision of the enclosing rectangle would permit the drawing of a second set of diagonals. The possibilities of this operation are certainly very great (G. V. Figs. 6a, 6b, p. 80; Fig. 12, p. 85; Fig. 14a, p. 87; Fig. 17, p. 99; Fig. 21, p. 102). Hambidge diagrams consist of a few lines carefully chosen from many available.

Before proceeding to draw our final conclusions relative to the "first and second claims" it will be necessary to touch briefly on three topics, the first being "static symmetry." The term covers two distinct types of design, those which are generally rectangular in shape, as vases and temple plans, and in addition distinguished by having rational proportions, and those arranged about a center (D. p. 10; Fig. 2, p. 104; G. V. p. 7; Fig. 5, p. 127; Chapter 12). Designing by rectangles is not suited to laying out circular and elliptic ornament arranged about a center, but is work of that kind any the less worthy when well done? Surely a rose window is not necessarily of a lower type than an arched one. The interior of a decorated dome with a circular mosaic design in the pavement below, certainly need not stand lower in the order of things artistic than a rectangular ceiling with corresponding pavement. In what follows we are alone concerned with rectangular static designs.

The second topic is a variety of analysis illustrated by Fig. 1. The enclosing rectangle of this kylix has the ratio 3.1148 and Mr. Hambidge uses (D. p. 116-117) 3.1180 (see Fig. 9) in his analysis, this being 0.0032 larger. Subtracting the difference from 3.1148 gives 3.1116. Hence any ratio between 3.1116 and 3.1180 will be as close a fit as that given in *The Diagonal.* The ratios of the above table were selected with this in view, the second ratio being that of the kylix without handles (Fig. 14 was used in D.). The remainder of the analysis (see our description above and D. pp. 116-117) depends on the construction of squares and the drawing of diagonals. Hence, any of the root-five values from the table may be used, and seven

times six or forty-two analyses will result, each differing from every other in some respects. For none of these will the errors much exceed those of the Hambidge analysis (which is included) and most of them will be more exact. In the same way there are eleven times eleven, that is one hundred twenty-one analyses by the root-thirteen system, and seventy-two by the rational system. We have then the remarkable example of a design which has dynamic symmetry of two kinds (root-five and root-thirteen) while in addition it has static symmetry.

The third topic relates to the several systems of rectangles. In preparing this paper a study of the root-five, root-thirteen, and a rational system was made, as already mentioned. For each of the systems of rectangles depending on the square roots of two, three, seven, eleven, and seventeen, elementary rectangles are selected and the relations between them determined. It does not appear that these, the root-thirteen, nor the rational system are markedly inferior to the root-five system as instruments of analysis. We cannot agree with the statement that the root-two and root-three systems are inferior to root-five (G. V. p. 102; Note VIII, p. 159). The former have never been provided with a selection of elementary rectangles comparable with the latter nor received similar development. In *Dynamic Symmetry*: *The Greek Vase* and the first eight issues of *The Diagonal* there are mentioned about five rectangles of the root-three system, twenty of the root-two, and one hundred nine of the root-five. It is interesting to note that these numbers are closely proportional to the percentages of Greek designs which are said to fall under the different systems (D. p. 87).

Nor are the above all that Greek geometry has to offer —designs might be built up of the square, the root-three, and the root-five rectangles as elements, our author to the contrary notwithstanding (D. p. 114; G. V. Fig. 12, p. 110; Note VIII, p. 159). Again, there are rectangles from irrationals of higher type than the square roots—"super-dynamic," as one might say. Some of them, the enclosing rectangles of the regular pentagon and others from the same source (not those of G. V. Chap. 3, whose width is determined by the circumscribing circle) would have been dear to the heart of any Pythagorean. But would not all

this, we are asked, require deep knowledge of geometry? Not at all. An artist with little knowledge except a few elementary constructions could have built up rectangles for a vase design, which would forever after defy discovery, from the vase itself.

From the above arguments we conclude as follows:

First. Any vase or other rectangular design can probably be analyzed, to an accuracy within the errors of construction, by each and every one of the root and rational systems of rectangles, did we but have complete and detailed information about them.

Second. It is practically impossible to prove that any one particular design, say Fig. 5b, p. 53 of *The Diagonal*, does not belong to some one system, the root-three for example, so enormous is the number of diagrams one would have to examine. Not a single demonstration has been given in support of the "second claim." Again to prove a design to be root-five one must needs prove it is not anything else, a still more hopeless task and one our author does not attempt.

Third. The classification of rectangular designs into static and dynamic, and the latter again into root-two, root-three, etc., as distinctions of form, probably has no basis in fact. The numerous Hambidge analyses are no proof of the classification. Even were it contended that such differences of form actually do exist, the separate types could not express æsthetic differences, because one is powerless to determine by inspection the character of the analysis and hence the type of the object (Langfeld, *The Aesthetic Attitude*, pp. 227-228). A modern silver bowl from say a root-three design could be duplicated very closely by designs in any of the other systems, in other words its type would be ambiguous.

Fourth. Whether the Greeks did or did not employ dynamic symmetry cannot, we believe, be proven by analyses after the Hambidge manner.

The Third Claim.

To place dynamic symmetry on a secure foundation appeal is made to nature—the human skeleton, the maple leaf, the sunflower are called upon to give it character. Let us examine the witnesses. In several articles in *The*

Diagonal (pp. 5, 27, 48, 71, 96, 118) measurements of two human skeletons are given, which, it is maintained, demonstrate the human skeleton to have dynamic symmetry of the root-five variety. As a matter of fact, the measurements neither do nor can show anything of the kind. The distinction between commensurable or rational quantities and incommensurable or irrational is one of the most fundamental and important that mathematical theory recognizes, but the distinction belongs to theory not to practice (H. E. Hawkes, *Advanced Algebra,* 1905, p. 53). The most accurate of measuring operations is powerless to distinguish the one kind of quantity from the other. Hence, when we are told (D. p. 8) certain ratios, found by dividing measured dimensions of a skeleton, are "never ending fractions" what is one to infer? Simply that this is an assumption, a postulate which rests on no demonstration. When later these ratios are identified with certain from the root-five system, an assumption is again made, namely that the ratios from the skeleton exactly equal those from the root-five system. We have no means of knowing whether they do or not. We conclude then that the articles above mentioned do not prove the skeleton to be of root-five proportions, they assume it.

It may be asked does not the same argument apply to the measurement of a Greek vase; is it not assumed to be root-two or root-five, as the case may be, without proof? This is indeed the fact, but with this difference—there is a degree of reasonableness in the supposition as applied to vases which is quite lacking in the case of any natural form. The vase was made by a Greek and much is known of their manner of living and thinking. They developed geometry and doubtless applied it in many ways to architecture and design. Whether they used dynamic symmetry is open to question, but it is at least possible. That root-five has any significance for the human skeleton is mere guesswork. Greek geometry has many possibilities to offer in the way of schemes of analysis, as we have seen, but for natural objects its restrictions are no longer pertinent. What nature's designer may have used is unknown—the possibilities are endless.

Again, as we have shown, probably any object can be analyzed by the root-five system, to a fair degree of ac-

curacy. Hence, to say that a vase and skeleton have some qualities of form in common, because they both belong to the root-five system, is like finding a relation between the architecture of the Woolworth Building and the New York Post Office by virtue of their both being laid out with a foot rule. Our author could hardly be defended by saying that what he really determines is that certain specific proportions are found in both the skeleton and Greek vases. In the first place, the way in which the proportions occur in the two would not in the least lead one to suspect any connection between them, especially as it is the living form and not its framework with which, as a rule, one becomes familiar. Even were there any æsthetic value in the proportions of the skeleton, one would not expect it to pass to the vase under the circumstances. Further, human skeletons vary greatly in their proportions and only rough averages are at all representative. These would be quite out of harmony with the exactness and incommensurability which distinguish dynamic symmetry.

Next comes the maple leaf whose form "strikingly resembles a regular pentagon." To an examination of its trussing is credited the discovery of dynamic symmetry in nature (G. V. p. 30). Be that as it may, the rectangular subdivisions of the regular pentagon (G. V. Chapter Three) bear no resemblance to the serrated edge and internal structure of the leaf. It is interesting to note that the widths of the several rectangles obtained are determined not by the width of the pentagon but by that of its circumscribing circle. This results in a curious paradox. The root-five system is represented to be based on the maple leaf, that is, the regular pentagon, but when one calculates the rectangle enclosing the pentagon, and subdivisions of this determined by its vertices, they are found not to belong to any of the root systems, depending, as before remarked, on higher irrationalities. One cannot (exactly) analyze the regular pentagon by the root-five system. One wonders about the hundreds of other leaf forms of which nothing is said.

Lastly, there is the sunflower (D. pp. 2, 45). The lines separating its seeds are logarithmic spirals arranged in two sets. Those of one set are congruent curves winding to the right, those of the other are likewise congruent

among themselves but wind to the left. Pine cones exhibit an analogous structure. The interest centers on the ratio of the number of curves in one set to that in the other. Each sunflower furnishes but one ratio but different flowers have different ones. These curiously, with but few exceptions, belong to an infinite series of fractions, namely $5/8$, $8/13$, $13/21$, $21/34$.......which have appeared in various mathematical investigations of the past (G. V. pp. 152-157). The successive fractions in the series have values which more and more nearly equal the ratio of the "golden section" or the ratio of the rectangle of the whirling squares of Mr. Hambidge. Thus is the connection with dynamic symmetry established. Without in any way wishing to belittle the scientific value and interest of these facts for botany, it is difficult to understand how any one could hope to have any emotional response, which the flower might produce in the beholder, carried over to the vase through any such long and intricate mathematical argument. The ratio for any one flower is not based on any conspicuous features of its form, nor does it depend on the nature of the curves separating the rows of seeds, only on their number. Further, the connection cannot be established by one flower, one must have a series of all sizes, and even then assume that were they to grow to unlimited size the ratios would follow the law of the above series. We doubt whether this or other varieties of phyllotaxis teach any lesson with regard to the ratios of value to art, but if they do, it must certainly be—"Use rational ratios"—"Make static designs."

However it may fare with the above details, the real essence, the great secret of dynamic symmetry has still to be considered; namely, the sides of the rectangles, the lines of the diagrams, though incommensurable are "commensurable in square," also "'dynamic symmetry deals with commensurable areas" (D. pp. 14, 48; G. V. Note III, p. 145, Note VI, p. 157). Our author explains that if squares be constructed on two adjacent sides of a rectangle and their areas are found to be commensurable, then the two sides of the given rectangle are "commensurable in square." In algebraic terms this means simply that the ratio of the rectangle in question is the square root of an integer or fraction. Thus, in the root-five sys-

tem, the sides of the root-five rectangle are commensurable in square, as are also the sides of five or six others that have been used in analyses, whose ratios are multiples of the square root of five. The property is not however true of the rectangle of the whirling squares, nor of nine-tenths of the rectangles used in the Hambidge diagrams.

If the statement "dynamic symmetry deals with commensurable areas" means "commensurable in square" there is nothing more to be said. If it means what it says, it is no more correct than the other. For example, if the height of Fig. 9 be unity, each S has area equal to one, each R equal to $\sqrt{5} \div 4$, and the area of the whole is $(4+\sqrt{5}) \div 2$. No one of these areas is commensurable with any other. We remark in passing that all rational rectangles are both commensurable in square and in area. Even were the author's statements not incorrect, no aesthetic qualities can rest on the distinction between commensurable and incommensurable. The most accurate measurement fails to separate the one from the other, what hope is there to do so by inspection? Further, in the case of vase designs, the areas in question are not present in the design itself, they pertain to the rectangular scaffolding by whose aid, it is assumed, the design was laid out.

The Fourth Claim.

It will not be necessary to discuss the relative merits of Greek and other art, the Gothic for example. Each quite accurately expresses the environment, life, thought, and aspirations of its creators, and though both are part of our artistic heritage, the art of this age, in so far as it is a spontaneous expression of present day conditions, cannot duplicate the past. In particular, we cannot hope to carry over into modern art any of the excellencies of the art of the Greeks by the employment of dynamic symmetry, even overlooking the absence of proof that it was ever used. Our previous argument has shown that probably any design admits of classification under all the several dynamic types and the static as well. A design having been analyzed in the Hambidge manner, does not on that account, possess any special excellence of form or of artistic qualities. The same is true of any modern de-

sign laid out by dynamic symmetry, it may prove to be quite commonplace or have exquisite beauty, just as might result from the employment of other methods.

There is practically nothing in *Dynamic Symmetry: The Greek Vase* and *The Diagonal* relative to the procedure to be followed in creative design, but the deficiency has in part been made good by enquiry among designers using the method. In the first place it is evident that dynamic rectangles, say those of the root-five system, are quite as inert and dead as are door-nails or roofing slates. We ask them in vain whether the head-board of a bed should be higher than the foot or not; whether a pitcher should be twice as broad as high or the reverse. They tell nothing. We are informed they are not expected to. First, one must know the kind of article he is to design, the service for which it is intended, the period and style to which it belongs, its general size and shape, the material of which it is made and the technique to be employed. The design as thus blocked out still admits a limited measure of freedom in the selection of final dimensions and proportions—these dynamic symmetry is called upon to determine. A diagram made up of dynamic rectangles is devised to harmonize with the blocked out design in such a way that none of the variations of size and shape which it permits are overridden. From this diagram the final dimensions are determined. That there may be several such diagrams either of dynamic or static rectangles of several types seems not to have attracted attention. Nor does it seem to have occurred to the users of the method that the forms one may thus determine are so extremely numerous that they fail to be characterized by any remarkable properties, and might just as well be obtained by mere caprice or the throwing of dice. The selection of one figure from the hundreds geometry has to offer is strikingly like the last mentioned procedure.

It is difficult to understand how the followers of the method—now numerous and lacking neither faith nor enthusiasm—can credit a collection of simple rectangles with having occult power to decide the last subtle gradation of proportion necessary to the production of a masterpiece, be it a pottery vase, a silver bowl, a marble statue, or a figure composition (D. pp. 121, 133-138, 153, 155-161).

Summary of Conclusions.

The analyses of Mr. Hambidge do not in any way constitute a proof that vase designs and others not arranged about a center admit—as to form or æsthetic significance—of being classified into static and dynamic types. The proof could not be carried through without an almost endless examination of thousands of constructions, and in the end there is little doubt that each and every design would be found to belong to all classes at once. That the Greeks ever employed dynamic symmetry does not seem capable of proof by geometry. The making of further analyses of the kind already published will not help the situation.

The claim that dynamic symmetry in any way expresses the essentials of plant and animal forms is without rational foundation. The statements that the diagrams of dynamic symmetry are "commensurable in square" and composed of commensurable areas, are for the most part incorrect. The attempt to base differences of artistic quality on the distinction between rational and irrational quantities, whether of length or area, is bound to fail—the eye is powerless to make the distinction.

The rectangles of dynamic symmetry are of themselves inert and lacking of any directive force. They stand ready, as do the rational rectangles, to be selected for such service as the intelligence of the designer may elect. As a method for modern designers, dynamic symmetry has nothing of value to offer, and by imposing false standards and needless restrictions can but hamper the freedom of creative inspiration.

Note. When the above paper was in process of publication my attention was called to one with identical title by Prof. Rhys Carpenter, which had but recently appeared in the American Journal of Archaeology, Vol. XXV, 1921, pp. 18-36. The two papers are, fortunately, not to any great extent duplicates, but rather each supplements the other, and their conclusions, in so far as they treat of the same phase of dynamic symmetry, are in substantial agreement. This is the more interesting since each was produced independently of the other, the one by an archaeologist with an interest for mathematics, the other by a mathematician attracted to art. Prof. Carpenter confines his attention to the archaeological questions involved while I have, in addition, discussed the reputed foundations of dynamic symmetry in nature, and its value to designers of the present day. Prof. Carpenter has done well to insist that note be taken of the "complete irrelevance of these (dynamic) rectangles to the actual areas of the vase, and especially to the contour curves which are so largely the animating life of the ancient vase" (p. 36), and by showing much simpler methods of design which may have been used. Ratios need not have been thought of at all by the Greek potters, the dimensions of the several parts of a design would have sufficed.

New York, April 17, 1921

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